Static and dynamical responses of neutron systems

From ultra-cold atoms to nuclear matter

Antoine BOULET

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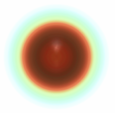
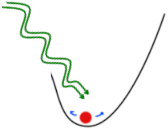
1. Motivation: First *ab-initio* calculation of static properties for neutron matter (NM) [Buraczynski and Gezerlis, PRL **116** (2016)]

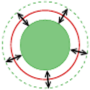
2. DFT based on low energy constants



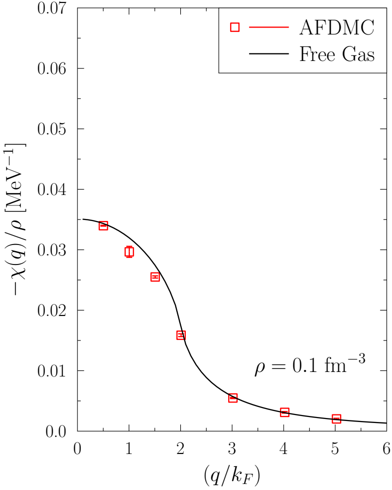
3. Static properties of cold atoms and neutron matter
   - Thermodynamical ground state properties
   - Static linear response



4. Dynamical properties of cold atoms and NM
   - Hydrodynamical regime and collective modes



Linear response of neutron matter: recent AFDMC calculation



First *ab-initio* calculation (AFDMC) of the linear response for neutron matter

Surprising results: close to free Fermi Gas response

Provide a **strong constraint** for functional theory:

- effective mass m^*
- compressibility $\kappa = -\chi(q=0)/\rho^2$

AFDMC: [Buraczynski and Gezerlis, PRL **116** (2016)]

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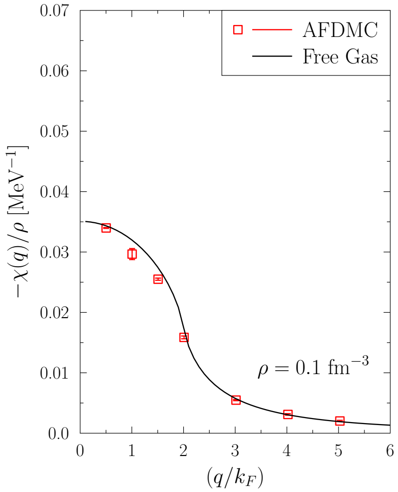

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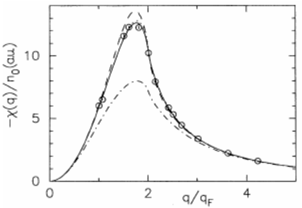
For comparison: electron gas response
[Moroni *et al.*, PRL **75** (1995)]



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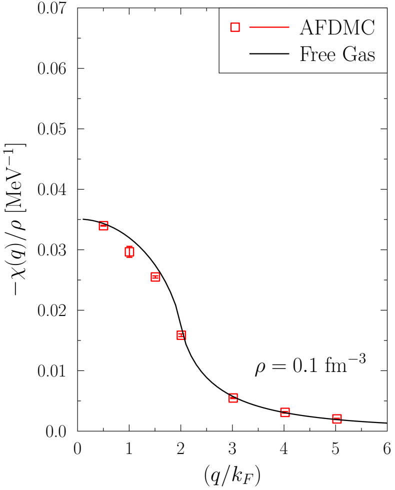

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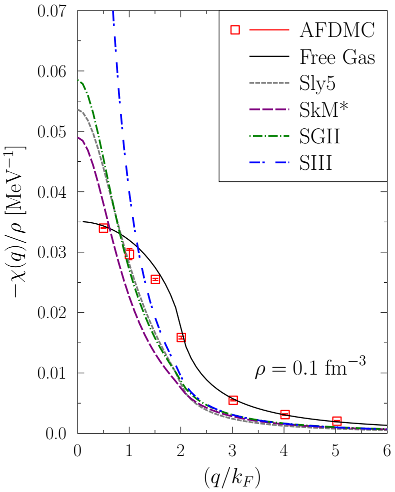
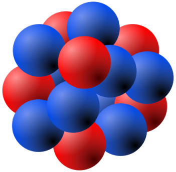


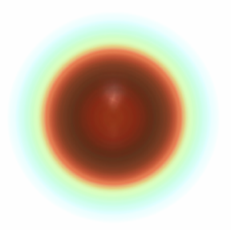
Skyrme functionals do not reproduce the response of neutron matter

- we use our non-empirical functional and tested it against AFDMC

AFDMC: [Buraczynski and Gezerlis, PRL **116** (2016)]

New Functional
based on low energy constants
(reminder)





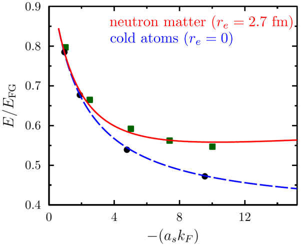
New type of functional without free parameter: short reminder

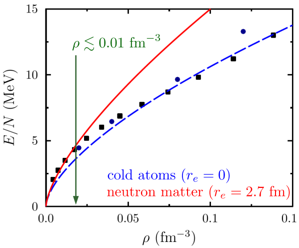
Non-empirical functional

[Lacroix, PRA **94** (2016)]
[Lacroix, **A.B.**, Grasso and Yang, PRC **95** (2017)]

$$\frac{E}{E_{FG}} = \xi(a_s k_F, r_e k_F)$$

$$= 1 - \underbrace{\frac{U_0}{1 - (a_s k_F)^{-1} U_1}}_{\text{zero-range part}} + \underbrace{\frac{R_0(r_e k_F)}{\left[1 - R_1 (a_s k_F)^{-1}\right]\left[1 - R_1 (a_s k_F)^{-1} + R_2(r_e k_F)\right]}}_{\text{effective range part}}$$





- ● [Gezerlis & Carlson, PRC (2010)]
- ■ [Carlson *et al.*, PTEP (2012)]
- ● [Akmal & Pandharipande, PRC (1998)]
- ■ [Friedman & Pandharipande, NPA (1981)]

Ground State (GS)
Thermodynamical properties

Some GS thermodynamical quantities

$$\frac{E}{E_{FG}} = \xi(a_s k_F, r_e k_F) \qquad (\text{FG : Free Gas})$$

$$P \equiv \rho^2 \frac{\partial E/N}{\partial \rho} \qquad \frac{1}{\kappa} \equiv \rho \frac{\partial P}{\partial \rho} \qquad \mu \equiv \frac{\partial \rho E/N}{\partial \rho} \qquad \rho = \frac{k_F^3}{3\pi^2}$$

Pressure P

$$\frac{P}{P_{FG}} = \xi + \frac{k_F}{2}\frac{\partial \xi}{\partial k_F}$$

Chemical potential μ

$$\frac{\mu}{\mu_{FG}} = \xi + \frac{k_F}{5}\frac{\partial \xi}{\partial k_F}$$

Compressibility κ

$$\frac{\kappa_{FG}}{\kappa} = \xi + \frac{4k_F}{5}\frac{\partial \xi}{\partial k_F} + \frac{k_F^2}{10}\frac{\partial^2 \xi}{\partial k_F^2}$$

Sound velocity c_s

$$\left(\frac{c_s}{c}\right)^2 = (m\rho\kappa)^{-1}$$

Cold atoms results ($r_e = 0$) near unitary

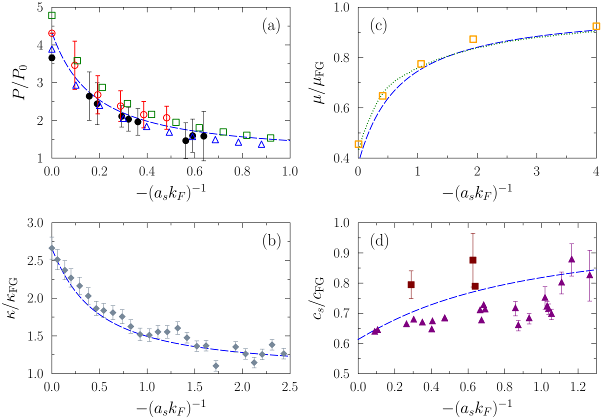
Survey of experimental and theoretical data

Theories

- ○ [Bulgac *et al.*, PRA **78** (2008)]
- □ [Haussmann *et al.*, PRA **75** (2007)]
- △ [Hu *et al.*, Europhys. Lett. 74 (2006)]
- □ [Pieri *et al.*, PRB **72** (2005)]
- ... [Astrakharchik *et al.*, PRL **93** (2004)]

Experiments

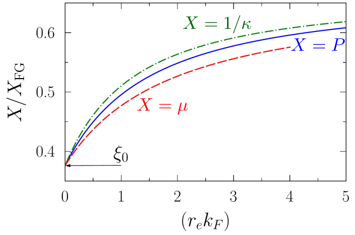
- ● [Navon *et al.*, Science **328** (2010)]
- ◆ [Navon *et al.*, Science **328** (2010)] [Ku *et al.*, Science **335** (2012)]
- ■ [Weimer *et al.*, PRL **114** (2015)]
- ▲ [Joseph *et al.*, PRL **98** (2007)]



In general the non-empirical DFT **works very well in cold atoms** at unitarity and away from unitarity.

Effective range effect

r_e – dependence at unitarity ($a_s \to -\infty$)



Neutron matter prediction ($a_s = -18.9$ fm and $r_e = 2.7$ fm)

$$\xi(r_e k_F) = \xi_0 + \frac{(r_e k_F)\eta_e^2}{\eta_e - (r_e k_F)\delta_e}$$

$$\simeq \xi_0 + (r_e k_F)\eta_e + (r_e k_F)^2 \delta_e + \cdots$$

Strong effective range dependence ($\simeq 50\%$)

[**A.B.** and Lacroix, arXiv:1709.05160 [nucl-th] (2017)]

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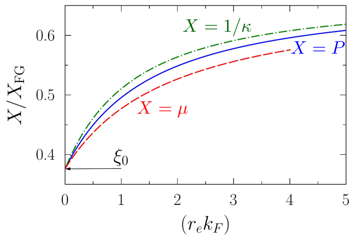

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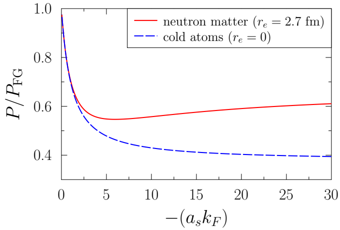

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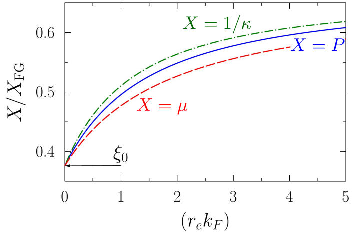

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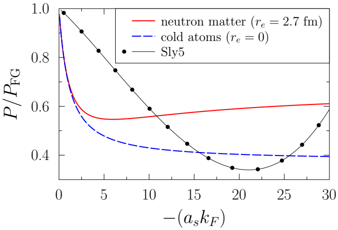

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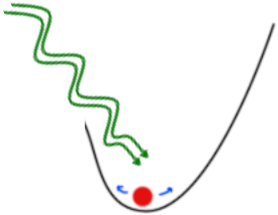
Static linear response



Linear response theory

RPA formalism for infinite matter

$$E = \int d\mathbf{r} \Big(\underbrace{\mathcal{K}[\rho(\mathbf{r})]}_{\text{kinetic}} + \underbrace{\mathcal{V}[\rho(\mathbf{r})]}_{\text{interaction}} \Big)$$

External field

$$\hat{V}_{\text{ext}} = \sum_j \phi(q,\omega) e^{i\mathbf{q}\cdot\mathbf{r}_j - i\omega t}$$

Response function χ

$$\rho(\mathbf{r}) \equiv \rho \to \rho + \delta\rho$$

$$\delta\rho = -\chi(q,\omega)\phi(q,\omega)$$

$$\chi = \chi_0 \left[1 - \frac{\delta^2 \mathcal{V}}{\delta\rho^2} \chi_0 \right]^{-1}$$

Static response function

$$\chi(q) = \lim_{\omega\to 0} \chi(q,\omega)$$

Compressibility sum-rule

$$\lim_{q\to 0} \chi(q) = -\rho^2 \kappa$$

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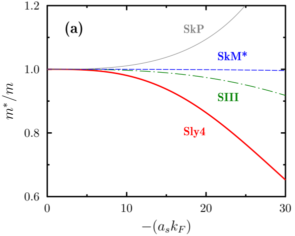
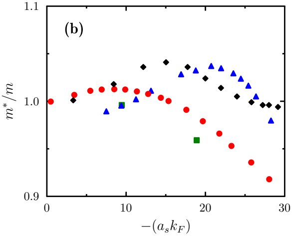
One difficulty: effective mass

Skyrme functionals



ab-initio calculations



- ▲ [Schwenk *et al.*, NPA **713** (2003)]
- ● [Wambach *et al.*, NPA **555** (1993)]
- ■ [Friedman *et al.*, NPA **361** (1981)]
- ◆ [Drischler *et al.*, PRC **89** (2014)]

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Linear reponse in cold atoms ($r_e = 0$)
Comparison with SLDA and Bersch parameter estimation

SLDA: Superfluidity Local Density Approximation (Bulgac *et al.*)

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- Bertsch parameter ξ_0 (α,β)
- effective mass m^* (α)
- pairing gap Δ (γ)

m^* and Δ seems to not affect too much the linear static response

[**A.B.** and Lacroix, arXiv:1709.05160 [nucl-th] (2017)]
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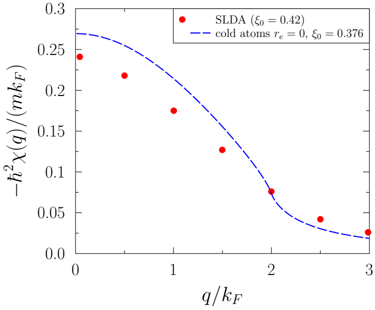

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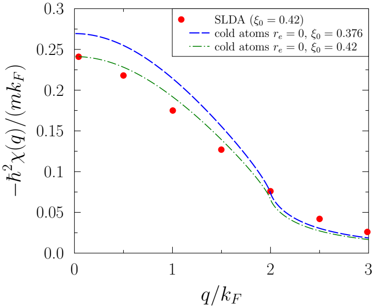
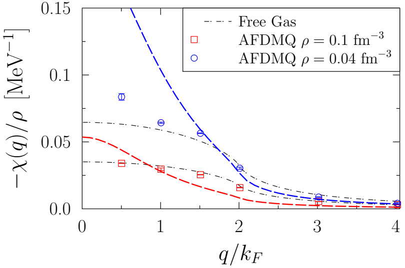


Linear static response function for neutron matter ($r_e = 2.7$ fm)
Comparison with recent QMC calculation

Empirical functional (Sly5)



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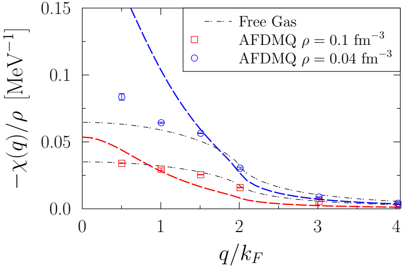
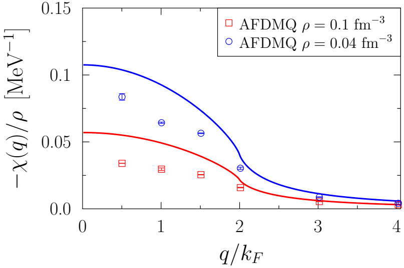


Non-empirical functional



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Adding p-wave
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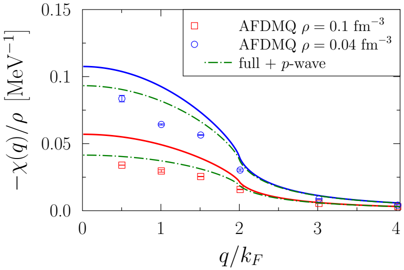
Non-empirical functional + p-wave



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AFDMC match Free Gas response = compensation effect of many contribution?

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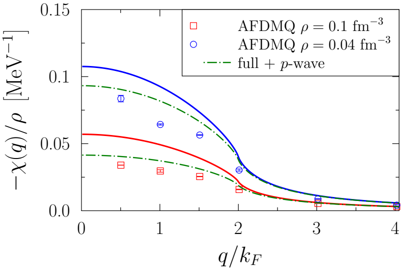

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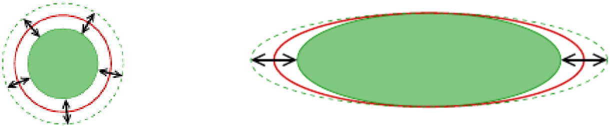
Dynamical response: hydrodynamical regime



Collective modes in trapped Fermi systems

Anisoptropic trap

$$U(\mathbf{r}) = \frac{m\omega_0^2}{2}\left(x^2 + y^2 + \lambda^2 z^2\right)$$

Hydrodynamical regime at equilibrium

$$\nabla^2 P = -\frac{1}{m}\nabla \cdot [\rho \nabla U]$$

Polytropic EoS

$$P \propto \rho^\Gamma \quad \text{with} \quad \Gamma = \kappa P$$

Γ: adiabatic index of infinite system

Linearized $\rho \longrightarrow \rho + \delta\rho e^{i\omega t}$

$$-m\omega^2\delta\rho = \nabla\cdot[\delta\rho\nabla U] + \nabla^2\left[\frac{dP}{d\rho}\delta\rho\right]$$

Solution of cigar-shaped / prolate ($\lambda \ll 1$):

$$\frac{\omega^p_{\text{rad}}}{\omega_0} = \sqrt{2\Gamma} \qquad \frac{\omega^p_{\text{ax}}}{\lambda\omega_0} = \sqrt{3-\Gamma^{-1}}$$

[Heiselberg, PRL **93** (2004)]

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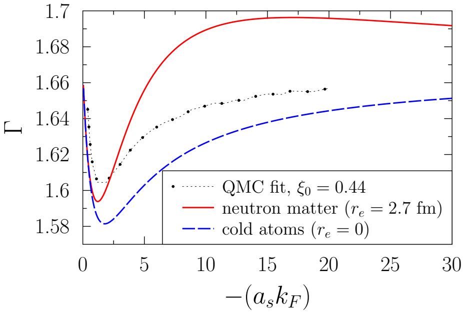

QMC: [Chang *et al.*, PRA **70** (2004)]

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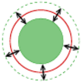
$$-m\omega^2 \delta\rho = \nabla \cdot [\delta\rho \nabla U] + \nabla^2 \left[\frac{dP}{d\rho}\delta\rho\right]$$

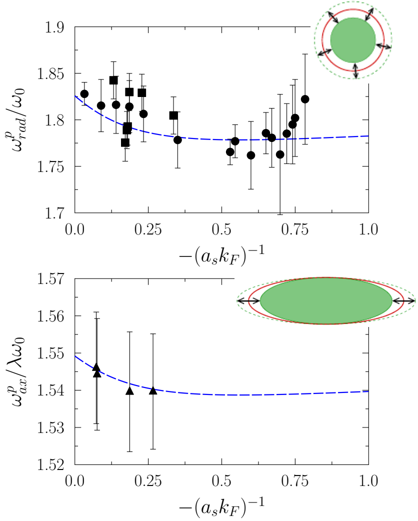
Solution of cigar-shaped / prolate ($\lambda \ll 1$):



$$\frac{\omega^p_{\mathrm{rad}}}{\omega_0} = \sqrt{2\Gamma} \qquad \frac{\omega^p_{\mathrm{ax}}}{\lambda\omega_0} = \sqrt{3 - \Gamma^{-1}}$$

[Heiselberg, PRL **93** (2004)]

Collective mode in trapped cold atoms ($r_e = 0$)



Linarized hydrodynamic + Polytropic EoS ($P = \rho^\Gamma$)

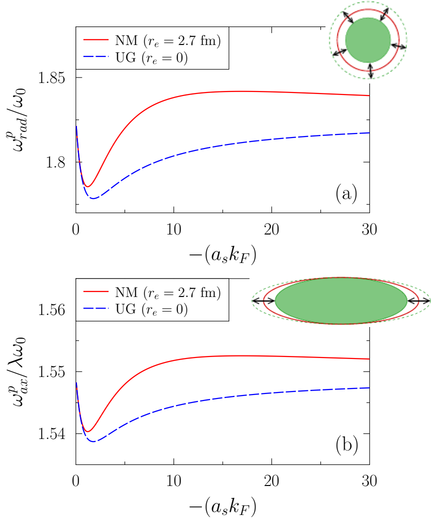
$$\frac{\omega^p_{\text{rad}}}{\omega_0} = \sqrt{2\Gamma}$$

$$\frac{\omega^p_{\text{ax}}}{\lambda\omega_0} = \sqrt{3 - \Gamma^{-1}}$$

- ▲ [Bartenstein *et al.*, PRL **92** (2004)]
- ● [Kinast, PRA **70** (2004)]
- ■ [Kinast, PRL **92** (2004)]

[**A.B.** and Lacroix, arXiv:1709.05160 [nucl-th] (2017)]

Collective mode in trapped neutron matter ($r_e = 2.7$ fm)



As for the GS thermodynamical properties and the static linear response, **Skyrme functional results are very different**

Exact calculations?

[**A.B.** and Lacroix, arXiv:1709.05160 [nucl-th] (2017)]

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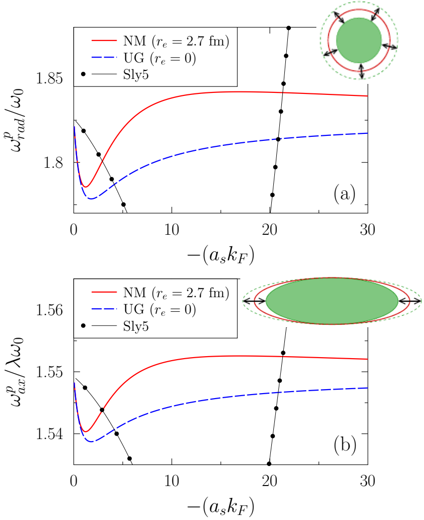

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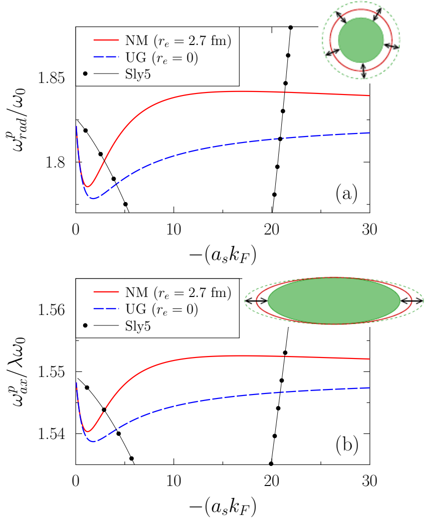

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Summary and perspectives

- A functional without free parameters was recently proposed and reproduce very well the properties of cold atoms
- The functional reproduce the *ab-initio* results at low density for neutron matter taking in account the effective range effect
- The static response reproduces reasonably AFDMC calculation for neutron matter
- The collective mode should be efficient to test and constrain the functional theories

Summary and perspectives

- **Short-term project**
  - Include the effective mass effect
  - Include the pairing in the functional
  - Application to finite Quantum Droplet (statics and dynamics)
  - Validity of ressumation to justify the functional
- **Long-term project**
  - Extend the theory to Symmetric Matter and finite nuclei
  - Study more precisely the BEC-BCS crossover

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$-\chi(0)/\rho = \rho\kappa$	$\rho = 0.04\ \mathrm{fm}^{-3}$	$\rho = 0.1\ \mathrm{fm}^{-3}$
Fermi Liquid	0.083	0.035
Lindhard (FG)	0.065	0.035
AFDMC	0.19	0.089
Neutron matter	0.108	0.057
Cold atoms ($r_e = 0$)	0.163	0.090

AFDMC: [Buraczynski and Gezerlis, PRL **116** (2016)]
[**A.B.** and Lacroix, arXiv:1709.05160 [nucl-th] (2017)]